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GENERALIZATION OF THE CLASSICAL RAYLEIGH EQUATION TO SEVERAL NON-NEWTONIAN LIQUIDS

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Equations are derived that describe the change in the radius of a spherical gas inclusion in the Bingham, Ellis, Reiner–Rivlin, Shul'man, Kapur–Gupta, and Oswald de Vielle non-Newtonian liquids, as well as in a power-law liquid.

The Rayleigh equation for highly viscous liquids with a finite relaxation time of elastic strains was obtained in [1]. In the present paper this equation is extended to non-Newtonian liquids, for which the rheological equations of state known to the present author are being extended.

In a spherical coordinate system the equation of motion, including strain and the continuity equation, are

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}\right)=-\frac{\partial P}{\partial r}+\frac{\partial\tau_{rr}}{\partial r}+\frac{2}{r}\tau_{rr}, \quad (1)$$

$$\frac{1}{r^2}\frac{d}{dr}(r^2v_r)=0, \quad (2)$$

if the bubble center is considered to be testing in the liquid. Integrating (2) with respect to r from the bubble radius R to infinity, we obtain the radial velocity of motion of the liquid $v_r = \dot{R}(R/r)^2$, expressed in terms of the drift velocity of the surface $\dot{R}$. Here and elsewhere, the dot over R denotes differentiation with respect to time. Substituting v_r into Eq. (1) and integrating it with respect to r in the limits R – ∞, we obtain

$$\rho\left(R\ddot{R}+\frac{3}{2}\dot{R}^2\right)=P_R-P_\infty+\tau_{rr}|_{r=\infty}-\tau_{rr}|_{r=R}+2\int\limits_R^\infty\frac{\tau_{rr}}{r}dr, \quad (3)$$

where P_R and P_∞ are the pressures in the liquid an the bubble surface and at infinity, and ρ is the liquid density. As P_∞ one can take the external static pressure in the liquid. The relation between P_R and the static pressure in the bubble P_0 is established by the Thomson relation

$$P_R=P_0-\frac{2\sigma}{R}-\frac{4}{3}\mu_0\left(\frac{\partial v_r}{\partial r}-\frac{v_r}{r}\right)\Big|_{r=R}, \quad (4)$$

where σ is the surface tension of the liquid. As μ_0 for non-Newtonian liquids, one can take the slope of the stream curve for small shear stresses (see [2]). The relation between the components of the strain tensor τ_{ij} and the components of the velocity deformation tensor $\dot{e}_{ij}$; i.e., the rheological equation of state, depends on the type of specific non-Newtonian liquid.

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A Viscoplastic Bingham Liquid. For this liquid

$$\tau_{ij} = -\left(\mu - \frac{\tau_0}{\sqrt{\frac{1}{2}\dot{e}_{lm}\dot{e}_{ml}}}\right)\dot{e}_{ij} \quad \text{for} \quad \frac{1}{2}\left(\tau_{ij}\tau_{ji}\right) > \tau_0^2, \tag{5}$$

$$\dot{e}_{ij} \equiv \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) = 0 \quad \text{for} \quad \frac{1}{2}(\tau_{ij}\tau_{ji}) < \tau_0^2, \tag{6}$$

where τ_0 is the yield of the liquid. From here on summation is performed on repeated indices. Substituting (5) into (3), we obtain after some calculations the equation

$$\rho\left(R\ddot{R} + \frac{3}{2}\dot{R}^2\right) = P_0 - P_H - \frac{2\sigma}{R} - \frac{2}{3}\mu_0\frac{\dot{R}}{R} - \frac{4}{\sqrt{3}}\tau_0 \ln\frac{H}{R}. \tag{7}$$

To remove the divergence of the integral in (3), the upper limit of integration was replaced by the finite value $H \gg R$. As H, one can choose the width of the liquid layer.

A Power-Law Liquid.

$$\tau_{ij} = k\left|\frac{1}{2}\dot{e}_{ml}\dot{e}_{lm}\right|^{\frac{n-1}{2}}\dot{e}_{ij}, \tag{8}$$

where k and n are constants for a definite velocity interval, and

$$\rho\left(R\ddot{R} + \frac{3}{2}\dot{R}^2\right) = P_0 - P_\infty - \frac{2\sigma}{R} - 4\mu_0\frac{\dot{R}}{R} - 2\cdot 3^{\frac{n-1}{2}}\left(\frac{2}{3n} - 1\right)k\left(\frac{\dot{R}}{R}\right)^n. \tag{9}$$

An Ellis Liquid.

$$\tau_{ij} = \left(\mu_1 + \mu_2\left|\frac{1}{2}\dot{e}_{lm}\dot{e}_{ml}\right|^{\frac{\alpha-1}{2}}\right)\dot{e}_{ij}, \tag{10}$$

where μ_1 and μ_2 are the ordinary viscosity coefficients of shear flow and of transverse viscosity, and are constant for a certain velocity interval

$$\rho\left(R\ddot{R} + \frac{3}{2}\dot{R}^2\right) = P_0 - P_\infty - \frac{2\sigma}{R} - \frac{10}{3}\mu_1\frac{\dot{R}}{R} - 2\cdot 3^{\frac{\alpha-1}{2}}\left(\frac{2}{3\alpha} - 1\right)\mu_2\left(\frac{\dot{R}}{R}\right)^\alpha. \tag{11}$$

The equality $\mu_0 = \mu_1$ was used in deriving (11).

A Reiner–Rivlin Liquid.

$$\tau_{ij} = \mu_1^*\dot{e}_{ij} + \mu_2^*\dot{e}_{im}\dot{e}_{mj}. \tag{12}$$

In the general case $\mu_i^* = \mu_i^*(I_2, I_3)$, where $I_2 = \sum_i\sum_j (\dot{e}_{ij})^2$ is a quadratic invariant tensor of velocity deformations, and $I_3 = \det\dot{e}$. If $\mu_i^* = \text{const}$, then

$$\rho\left(R\ddot{R} + \frac{3}{2}\dot{R}^2\right) = P_0 - P_\infty - \frac{2\sigma}{R} - 4\mu_0\frac{\dot{R}}{R} + \frac{2}{3}\mu_1^*\frac{\dot{R}}{R} - 4\mu_2^*\left(\frac{\dot{R}}{R}\right)^2. \tag{13}$$

The Rayleigh equation for this liquid is easily obtained also in the more general case $\mu_i^* = \mu_i^*(I_2, I_3)$ if the explicit dependence of the functional dependence of μ_i^* on I_2 and I_3 is known.

A Generalized Viscoplastic Shul'man Liquid [3].

$$\tau_{ij} = 2\left(\frac{\tau_0^{\frac{1}{n}}}{A^{\frac{1}{m}}} + \mu^{\frac{1}{m}}\right)A^{\frac{n}{m}-1}\dot{e}_{ij}, \quad A = (2\dot{e}_{ij}\dot{e}_{ji})^{-\frac{1}{2}}, \tag{14}$$

where A is the intensity of velocity deformation

$$\rho\left(R\ddot{R}+\frac{3}{2}\dot{R}^2\right)=P_0-P_\infty-\frac{2\sigma}{R}-4\mu_0\frac{\dot{R}}{R}+\left(4-\frac{8}{3}\frac{m}{n-1}\right)\times$$
$$\times(2\sqrt{3})^{\frac{n-m-1}{m}}\tau_0^{\frac{1}{n}}\left(\frac{\dot{R}}{R}\right)^{\frac{n-1}{m}}+\left(4-\frac{8}{3}\frac{m}{n}\right)(2\sqrt{3})^{\frac{n-m}{m}}\mu^{\frac{1}{m}}\left(\frac{\dot{R}}{R}\right)^{\frac{n}{m}}. \quad (15)$$

A Nonlinear-Viscous Liquid [4].

$$\tau_{ij}=\Psi(I_1)B_{ij},\ B_{rr}=2\frac{\partial v_r}{\partial r},\quad I_1=4\left(\frac{\partial v_r}{\partial r}\right)^2+8\left(\frac{v_r}{r}\right)^2. \quad (16)$$

The Rayleigh equation is easily obtained by expression (3), having assigned an explicit form of the functional dependence $\Psi(I_1)$.

A Kapur–Gupta Liquid.

$$\tau_{ij}=\mu_1\dot{e}_{ij}+\mu_2(\dot{e}_{ij})^2+\mu_3(\dot{e}_{ij})^3+\ldots, \quad (17)$$

where μ_i = const. Confining ourselves to the third term in (17), we obtain the equation

$$\rho\left(R\ddot{R}+\frac{3}{2}\dot{R}^2\right)=P_0-P_\infty-\frac{2\sigma}{R}-4\mu_0\frac{\dot{R}}{R}+\frac{2}{3}\mu_1\frac{\dot{R}}{R}-\frac{8}{3}\mu_2\left(\frac{\dot{R}}{R}\right)^2+\frac{56}{9}\mu_3\left(\frac{\dot{R}}{R}\right)^3. \quad (18)$$

Here one can put $\mu_0=\mu_1$.

An Oswald de Vielle Liquid.

$$\tau_{rr}=2KI_2^{n-1}\frac{\partial v_r}{\partial r},\quad I_2=(2\dot{e}_{ij}\dot{e}_{ji})^{\frac{1}{2}}, \quad (19)$$

$$\rho\left(R\ddot{R}+\frac{3}{2}\dot{R}^2\right)=P_0-P_\infty-\frac{2\sigma}{R}-4\mu_0\frac{\dot{R}}{R}-4(2\sqrt{3})^{n-1}K\left(\frac{\dot{R}}{R}\right)^n. \quad (20)$$

The equations obtained can be used to determine the rheological constants in the non-Newtonian liquids considered, as well as in studying boiling processes on the basis of the mathematical model of growth of a gas bubble, suggested in [5].

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